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Optimizing Pile Group Design Using a Real Genetic Approach

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ABSTRACT

This paper presents a practical methodology for optimizing pile group design of bridge foundation. A code-based method is used to design and analyze piled foundation. Then, a mathematical model for optimizing the problem by a real genetic algorithm is established with the direct total cost of pile foundation as the objective function. The global optimal solution (GOS) is first found by the exhaustive searching method (ESM). The GOS is a base for verifying the performance of different searching methods. The real genetic algorithm (RGA) is then proposed to globally search the optimal solution in a much little time than ESM. Seven real design cases are used to test the performance of RGA. Static penalty method (SPM) and Death penalty method (DPM) are used to treat constrained functions in the RGA used in the study. The analysis results show that the RGA can obtain the solutions within 48-61 minutes which are only about 1/1,000 of the time spent by the ESM. The construction costs of those solutions only differ from those of the global minimum solutions by 1.62-2.0% in average, and the minimum cost one of the solutions is the same as the global minimum solution. Thus, the proposed optimization methodology is time-efficient and technically practicable to be used in daily engineering works.

INTRODUCTION

Before the global financial crisis of 2008, the prices of oil and construction materials amazingly inflated up from 2007 to the middle of 2008. This makes people really realize that the resource of earth is rather limited. For sustainable development on earth, people have to cherish the use of earth resource. Based on the concept, civil engineers should optimize their designs of infrastructures as saving as possible so that the use of concrete and steel can be greatly decreased and the green house gas emission can be significantly reduced accordingly. In addition, there are more and more construction projects were executed in Build-Operation-Transfer (BOT) contracts. Only designs considering both safety and economy are more competitive in the current construction market. The above considerations raise the importance and urgency of optimal design for engineering project.

Since Dantzig (1947) proposed simplex method for solving linear programming problems in military and industry, numerous optimization algorithms have been developed and applied to solve for various engineering problems, such as structural design, transportation planning and construction management. However, only a small amount of research has been devoted to optimizing pile foundation design. Chow and Thevendran (1987) have used pile length as the main design variable to minimize differences in bearing loads between the piles in the pile group. Hoback and Truman (1993) used the optimality criteria (OC) method to conduct least weight design for a steel pile group. Hurd and Truman (2006) introduced a weightless optimality rule into the original OC approach to treat design variables, (e.g., the spacing and battering of the piles,) that has no measurable effect on the objective function. They only used sectional size and battering angle as design variables. Huang and Hinduja (1986) adopted a quasi-Newton method to optimize the shape of a pile foundation with the assumption of a linear force-deflection relationship for the pile-soil system. Valliappan et al. (1999) applied the generalized reduced gradient method to optimize pile foundation design with the lowest cost objective. Their design variables included pile length, diameter, number and pile cap. The FEM method was used to analyze pile foundation. The allowable total and differential settlements were the only constraints. Kim et al. (2001, 2002) used recursive quadratic programming and genetic algorithm to optimize the layout of a pile foundation, with minimum differential settlement being the objective and with the assumption of linear pile-soil interaction. Ng et al. (2006) adopted a genetic algorithm to optimize the design of in-situ bored pile foundation groups using the total volume of pile concrete as the objective function without considering steel reinforcement. Chan et al. (2009) presented an automatic optimal design method using a hybrid genetic algorithm for pile group foundation design with the concrete volume of the piles and the cap as the objective function.

All the above research has contributed much to the problems of optimizing pile foundation design. However, most of these methods might be difficult to apply in practice owing to the fact that the adopted design method and constraints are not code-based and the objective function is not the total cost of the pile foundation system. Moreover, Global minimum solutions were not found to verify the performance of their proposed optimization algorithms. This study focuses on optimizing pile foundation design of a bridge structure using a code-based design method. The real genetic algorithm (RGA), a global search method, was used to find the optimal solution. Combining the design and searching knowledge, the authors present a practical methodology for optimizing pile group design. The following summarizes the design method and mathematical model, exhaustive and RGA search, performance of the RGA, and some concluding remarks.

Design method and mathematical model

1. Design method

For popular use, the adopted design method must be practical, not too complicated, and code-based. This study is based on the design code for bridge foundations published by the Japanese Road Association (JRA, 2002), supplemented by the Taiwan foundation and reinforced concrete design codes. The main design considerations are summarized and explained below.

1.1 Design variables

The layout of all pile foundations includes two assumptions. One is the equal diameter and length of all the piles and the other is the rectangular and symmetrical arrangement of pile locations. The design variables include pile length, pile diameter, thickness of the pile cap, pile spacing and pile numbers, in two directions, transverse and longitudinal to the bridge axis. All the variables are discrete real numbers except for pile numbers, which are integers. The symbols representing foundation size, applied force, and their directions, are shown in Fig. 1. The selection of the design variables has to comply with some limitations, such as the usable land area, maximum pile length due to piling capability and pile spacing in relation to excessive grouping and construction problems.

1.2 Factors considered in the pile design

The following main factors are considered in the pile design which include (1) Minimum thickness of pile cap; (2) pile bearing capacity; (3) group effect of pile foundation; (4) effect of soil liquefaction; (5) deformation and stress analysis of pile foundation; (6) resistance of pile cap; and (7) reinforced design. The minimum cap thickness is determined by checking the rigidity requirement of the pile cap, the punching shear requirement, and the connection type between pile and cap. The allowable compressive, uplift and lateral bearing capacities of a single pile have to be checked for normal and earthquake load cases.

The group effects of a pile foundation are taken into account through two group efficiency factors, η_v (Yang and Han, 1997) and η_h (Liu, 1992), for the axial and lateral loading directions, respectively. The bearing capacity and stiffness of a pile foundation is reduced through a reduction factor D_E owing to foundation soil liquefaction during earthquakes as suggested by JRA (2002). The deformation of pile cap and the internal forces in the individual pile was analyzed by the model shown in Fig. 2. In this model, the pile cap is assumed to be a rigid plate. The deformations of the cap, δ_x, δ_y, and α can be solved for using stiffness matrix equation under the applied forces H_0, V_0 and M_0 at the center of the cap bottom. The lateral resistance H' and moment resistance M' provided by the side of the cap are also considered, as shown in Fig. 3. The design of reinforced steel must follow the requirements of the Design Code and Commentary to Concrete Structure in Taiwan (DCCCST, 2005) which is a modified local version of the ACI code (318-02). Basically, the steel bar design of pile and pile cap uses an exhaustive search procedure to search out the least weight solution from available design combinations incorporating different bar sizes and spacing which are commonly used in engineering practices.

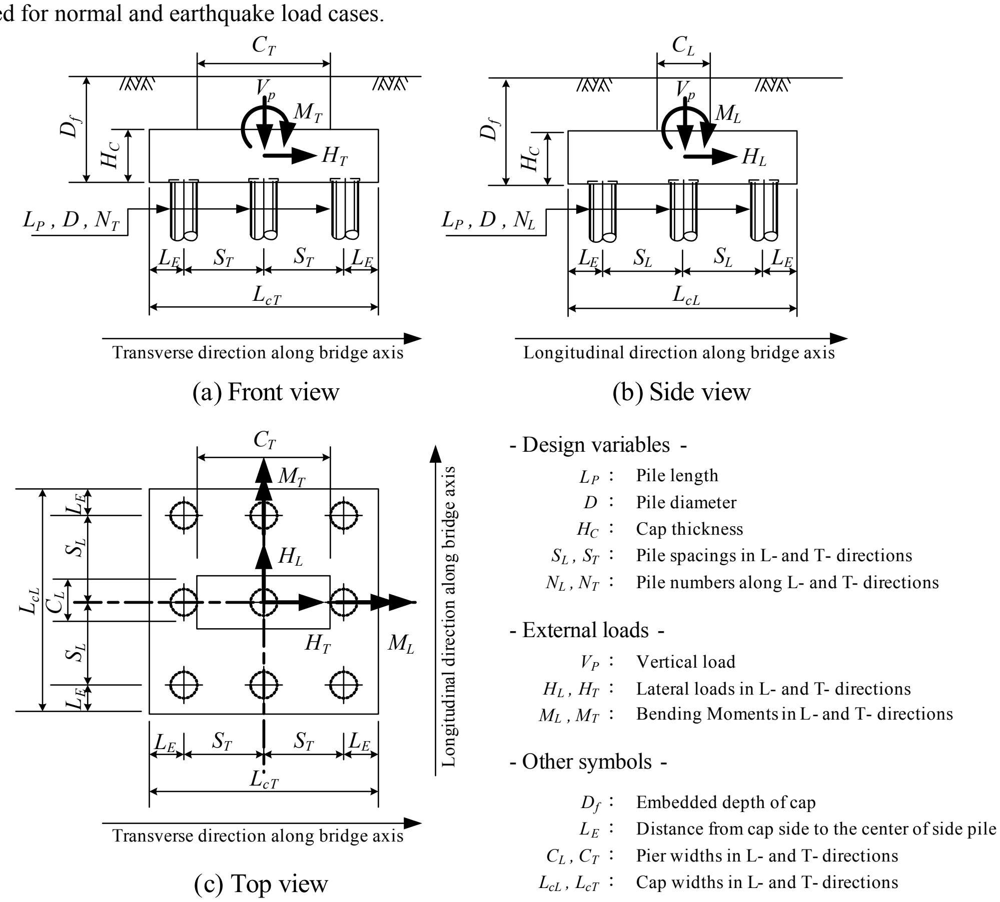

Fig. 1 Symbols for the studied piled foundation

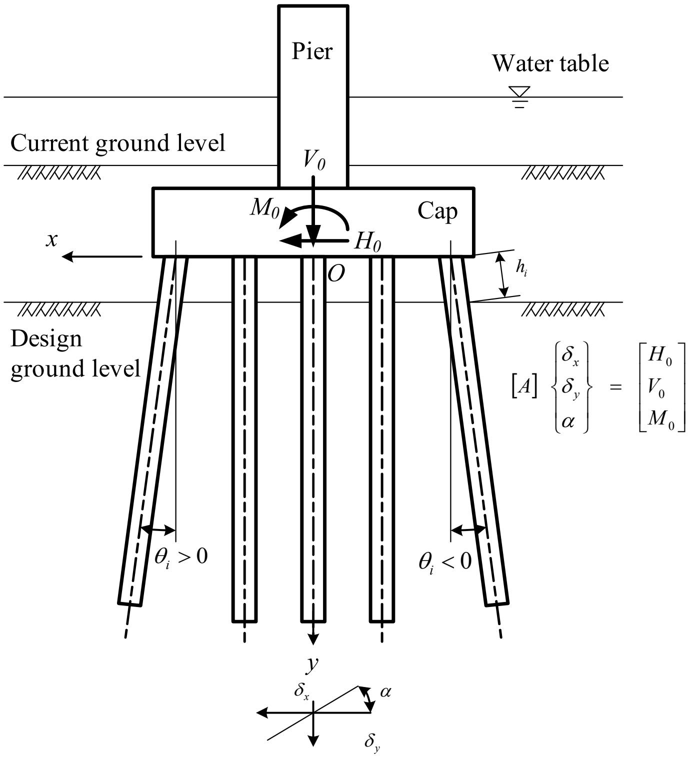

Fig. 2 Analysis model of the pile foundation examined

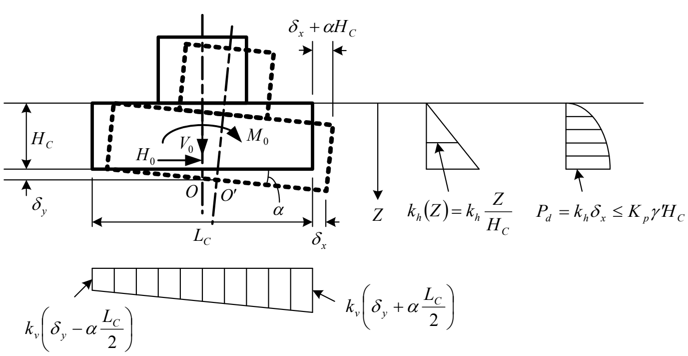

Fig. 3 Resistances at the sides and the bottom of the pile cap

2. Mathematical model for optimizing pile foundation design

2.1 Objective function

The total cost of a pile foundation is defined as the objective function $F(X)$:

$$F(X)=F_1(X)+F_2(X)+F_3(X)+F_4(X) \tag{1}$$

where X is a set of design variables, $X=(D, L_P, N_T, N_L, S_T, S_L, H_C)$; $F_1(X)$ is the excavation cost of the foundation pit; $F_2(X)$ is the cost of all the piles; $F_3(X)$ is the cost of the pile cap; $F_4(X)$ is the cost of backfilling the foundation pit. These will be described in further detail below.

2.1.1 Cost of excavation and backfill

The excavation cost is $F_1(X)=V_{cut}f_1$, where $V_{cut}(m^3)$ is the volume of the excavation; and $f_1(NTD/m^3)$ is the unit price of earth excavation. Fig. 4 shows a schematic diagram of the foundation pit. The pit is excavated from ground level to the bottom of the cap with a slope angle of 45°. The excavation bottom is the area of the cap plus 1 m on either side, for construction space. Thus, the volume of excavation can be written as

$$V_{cut}=L_{cL}L_{cT}D_f+D_f^{\,2}\left(L_{cL}+L_{cT}+2\right)+D_f\left(L_{cL}+L_{cT}+1\right)+\frac{4}{3}D_f^{\,3}) \tag{2}$$

where L_{cL} and L_{cT} are the widths of the pile cap in the longitudinal and transverse directions respectively; D_f is the embedded depth of the pile cap. The backfill cost is $F_4=V_{fill}f_6$, where $V_{fill}(m^3)$ is the volume of compacted backfill; and f_6 is the unit price of the compacted backfill. The volume of compacted backfill is shown in Fig. 5.

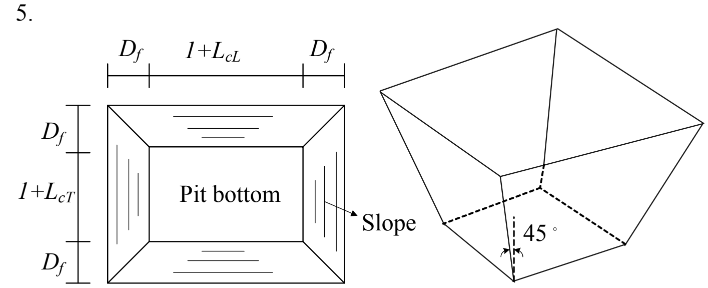

Fig.4 Schematic diagram showing earth excavation

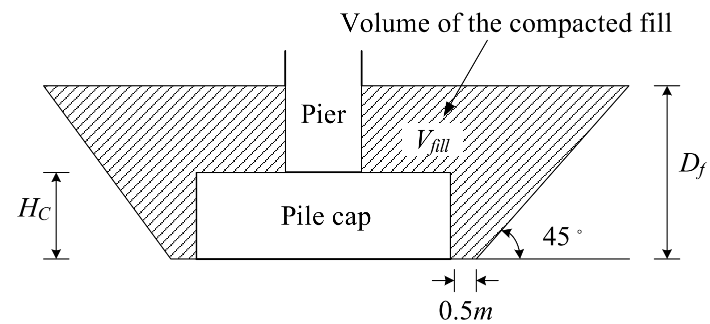

Fig. 5 Schematic diagram showing the volume of compacted fill

2.1.2 Cost of pile and pile cap

The cost of the pile consists of material costs for concrete and re-bar, and the cost for installing the pile. It can be written as

$$F_2(X)=N_LN_T\left(V_{pc}f_2+V_{ps}\gamma_sf_3+L_Pf_4\right) \tag{3}$$

where N_L and N_T are the numbers of the pile in the longitudinal and transverse directions to the bridge axis, respectively; $V_{pc}(m^3)$ is the volume of concrete in the pile; $V_{ps}(m^3)$ is the volume of steel in the pile; γ_s is the unit weight of the steel re-bar; $f_2(NTD/m^3)$ is the unit price of the concrete; $f_3(NTD/ton)$ is the unit price of the steel re-bar; and $f_4(NTD/m)$ is the unit price of pile installation. The cost of the pile cap includes the material costs for concrete and rebar, and the cost of formwork for casting the cap. It can be written as

$$F_3(X) = A_{cap} f_5 + V_{cc} f_2 + V_{cs} \gamma_s f_3 \tag{4}$$

where $A_{cap}(m^2)$ is the area of the formwork; $V_{cc}(m^3)$ is the concrete volume for the cap; V_{cs} is the steel volume for the cap; and $f_5(NTD/m^2)$ is the unit price for installing the formwork.

2.2 Constraints

There are a total of six constraints in the proposed mathematical model for optimizing pile foundation design. They are described in detail below.

2.2.1 Restriction of usable land

The size of the pile cap and excavation must be less than the area of land available for use. This constraint can be written as

$$g_1(X) = \frac{(N_i - 1)S_i + 2(D + D_f + 1)}{(L_{ci})_u} - 1 \le 0 \tag{5}$$

where N_i and S_i are the number of piles and the pile spacing in the i direction, respectively; $(L_{ci})_u$ is the upper limit of the cap size in the i direction.

2.2.2 Limitation of pile spacing

According to JRA (2002), the minimum pile spacing has to be greater than the larger one of 0.75 m and 2.5 times the pile diameter. The constraint can be written as

$$g_2(X) = \frac{\max(0.75, 2.5D)}{S_i} - 1 \le 0 \tag{6}$$

2.2.3 Restriction of pile length

The pile length is restricted by the construction ability of the piling machine. Assuming that the maximum pile length that the machine can install is $(L_P)_u$, the constraint can be written as

$$g_3(X) = \frac{L_P}{(L_P)_u} - 1 \le 0 \tag{7}$$

2.2.4 Limitation of shear resistance of pile cap

The shear resistance is mainly related to the thickness of the pile cap and can be designed by checking the requirements for punching shear and beam shear. The minimum required cap thickness $H_{C,req}$ is derived in Tab. 1 by (JRA, 2002 and DCCCST, 2005). The resulting constraint can be written as

$$g_4(X) = \frac{H_{C,req}}{H_C} - 1 \le 0 \tag{8}$$

2.2.5 Bearing capacity requirements

The compressive, lateral and tensile forces P_N, P_H and R_N applied to the pile head must be less than the allowable compressive, lateral and tensile bearing capacities P_a, H_a and R_a for the pile. The constraints can be written as

$$\left.\begin{aligned} g_5(X) &= \frac{P_N}{P_a} - 1 \le 0 \\ g_6(X) &= \frac{P_H}{H_a} - 1 \le 0 \\ g_7(X) &= \frac{R_N}{R_a} - 1 \le 0 \end{aligned}\right\} \tag{9}$$

2.2.6 Limitation of lateral displacement of the pile head

According to JRA (2002), the allowable lateral displacement δ_a at the pile head is equal to 1.0 cm in normal condition and 0.01D in earthquake condition (when $D \le 1.5m, \delta_a = 1.5cm$). Therefore, this constraint can be written as

$$g_8(X) = \frac{\delta}{\delta_a} - 1 \le 0 \tag{10}$$

Tab. 1 Formulae for determining cap thickness

<table>
<tr><th rowspan="2">Type of pile</th><th colspan="2">Type of pile head connection</th><th colspan="2">Cap thickness for punching shear requirement</th><th rowspan="2">Cap thickness for rigidity requirements</th></tr>
<tr><th>Pile head embedded into the cap</th><th>Connection using extended reinforcement into the cap</th><th>Working stress method</th><th>Ultimate strength method</th></tr>
<tr><td>Bored pile</td><td>-</td><td rowspan="4">$H_C \ge 0.1 + 35d_b + D/2$</td><td rowspan="4">Punching shear for pier
$H_{CE} \ge \dfrac{\sqrt{(C_L + C_T)^2 + \dfrac{4V'_P}{0.53\sqrt{f'_c}}} - (C_L + C_T)}{4}$
Punching shear for side pile
$H_{CE} \ge \dfrac{\sqrt{D^2 + \dfrac{4P_{V,\max}}{0.53\pi\sqrt{f'_c}}} - D}{2}$
Beam shear
$H_{CE} \ge \dfrac{q_c \max(L_{cL} - C_L, L_{cT} - C_T)}{2(q_c + 0.292\sqrt{f'_c})}$
$H_C = H_{CE}$ + thickness of protecting layer</td><td rowspan="4">Punching shear for pier
$H_{CE} \ge \dfrac{\sqrt{(C_L + C_T)^2 + \dfrac{4V'_P}{0.85 \times 1.06\sqrt{f'_c}}} - (C_L + C_T)}{4}$
Punching shear for side pile
$H_{CE} \ge \dfrac{\sqrt{D^2 + \dfrac{4P_{V,\max}}{0.85 \times 1.06\pi\sqrt{f'_c}}} - D}{2}$
Beam shear
$H_{CE} \ge \dfrac{q_c \max(L_{cL} - C_L, L_{cT} - C_T)}{2(q_c + 0.85 \times 0.53\sqrt{f'_c})}$
$H_C = H_{CE}$ + thickness of protecting layer</td><td rowspan="4">$H_C \ge \sqrt[3]{\dfrac{3K_V N_L N_T \bar{\lambda}^4}{L_{cL} L_{cT} E_c}}$</td></tr>
<tr><td>Steel pipe pile</td><td>$H_C \ge \dfrac{\sqrt{D^2 + \dfrac{4P_{V,\max}}{\pi[8.5 + (f'_c - 210)/60]}} + D}{2}$</td></tr>
<tr><td>Prestressed High Strength Concrete pile (PHC pile)</td><td>$H_C \ge \max(D + h, 50t + h)$
$h = \dfrac{\sqrt{D^2 + \dfrac{4P_{V,\max}}{\pi[8.5 + (f'_c - 210)/60]}} - D}{2}$</td></tr>
<tr><td>Precast Reinforced Concrete pile (RC pile)</td><td>$H_C \ge \dfrac{\sqrt{D^2 + \dfrac{4P_{V,\max}}{\pi[8.5 + (f'_c - 210)/60]}} + D}{2}$</td></tr>
</table>

Note： H_{CE} :Effective cap thickness（m） E_c :Young's modulus of cap concrete（kN/m^2） K_V :Spring coefficient at pile head（kN/m）

d_b :Diameter of rebar（m） f'_c :Compressive strength of concrete（kgf/cm^2） $\bar{\lambda}$:Cantilever length of pile cap（m）

t :Thickness of hollow PHC pile（m） $P_{V,\max}$:Maximum compressive load of pile（kN） q_c :Vertical surcharge on the cap（kN/m^2）

V_P :External vertical load（kN） $V'_P = V_P\left(\dfrac{\text{total number of piles - pile number under the pier}}{\text{total number of piles}}\right)$ $q_c = \dfrac{V_P}{L_{cL} L_{cT}}$

C_L 、C_T :Pier dimensions in longitudinal and transverse directions（m）

2.3 Safety surplus index

The safety surplus index (SSI) must be defined in order to understand the conservativeness of the analyzed stress and deformation states for a feasible design solution. The SSI for the i^{th} stress or deformation state is defined below.

$$S_i = \left(1 - \frac{V_{d,i}}{V_{all,i}}\right) \times 100\% \tag{11}$$

Where $V_{d,i}$ is the i^{th} analyzed variable state, such as the force and displacement of the pile head; $V_{all,i}$ is the allowable value of the i^{th} variable state required by code, such as the allowable bearing capacity and lateral displacement. This index denotes the normalized difference between the analyzed variable and the allowable value. When $S_i \geq 0$, it means the analyzed i^{th} state is safe. However, when $S_i < 0$, it indicates that unsafe condition prevail in the i^{th} state, which might imply that the design is controlled by the i^{th} variable state. This study proposed eight S_i indices. They are S_1 for checking compressive bearing capacity of the pile; S_2 for tensile bearing capacity of the pile; S_3 for compressive bearing capacity of the pile group; S_4 for lateral bearing capacity of the pile group; S_5 for lateral displacement of the pile head; S_6 for the punching shear check of the bridge pier; S_7 for the punching shear check of a side pile; and S_8 for checking the beam shear.

3. Exhaustive Search and Real Genetic Algorithm

3.1 Exhaustive Search Method (ESM)

For finding the global optima, an automatic pile analysis program was developed by the authors to perform exhaustive search. The solution space is approximated as the set of the combinations of all discrete design variables with an acceptable discretized size. The program carried out pile analyses on all the solutions in the space. The infeasible solutions were then removed out based on the analysis results, leaving a space of feasible solutions. The feasible solutions were sorted by the cost. Then, the global optima is defined as the one with the minimum cost.

3.2 Real Genetic Algorithm (RGA)

A genetic algorithm belongs to a class of adaptive stochastic optimization algorithms. The terminology of genetic algorithm was first used by Holland (1975). The basic idea is to try to mimic natural evolution process in order to find a good algorithm. A typical genetic algorithm requires (1) a genetic representation of the solution domain and (2) a fitness function to evaluate the solution domain. Its general procedure can be summarized as below.

1 Choose the initial population of individuals in some way
2 Evaluate the fitness of each individual in that population
3 Repeat the following steps on this generation until termination: (time limit, sufficient fitness achieved, etc.)
   a. Select the best-fit individuals for reproduction
   b. Breed new individuals through crossover and mutation operations to give birth to offspring
   c. Evaluate the individual fitness of new individuals
   d. Replace least-fit population with new individuals

A. Fitness function

There are a large number of different types of genetic algorithms. According to no lunch free theorem (Wolpert and Macready, 1997), the performance of an optimization algorithm is always problem-dependent. Thus, this research aims to understand if the genetic algorithm is suitable to finding the global optimal solution of piled foundation design problems and perform some sensitivity study of the parameters used in the genetic algorithm. The real valued encoding is used for its fitness to the problem. The chromosome is represented by the design vector of variables $X = (D, L_P, N_L, N_T, S_L, S_T, H_C)$ as defined before. The fitness function is defined as the linear combination of the objective function and the unified constraint function as below.

$$\text{Fitness Function} = F(X) + R_C Z(X) \tag{12}$$

In which $F(X)$ is the objective function, R_C is a penalty parameter and $Z(X)$ is the unified constraint function that is defined as

$$Z(X) = \left(\sum_{j=1}^{n_g} \max(0, g_j(X))^2 \right)^{1/2} \tag{13}$$

From its definition, $Z(X) \geq 0$. When $Z(X) > 0$, it means X is an infeasible solution. The smaller the $Z(X)$, the more close the X to the feasible zone. When $Z(X) = 0$, it means all the constraints are satisfied with and the X is a feasible solution.

There are a great number of methods to consider the influence of constraint function. However, the treatment is also problem-dependent. For simplicity, the simple static penalty method (SPM) and death penalty method (DPM) are used to reflect the influence of constraint function. The SPM uses a constant penalty parameter R_c with a value of 1×10^{15} which makes the term $R_C Z(X)$ of the same order with the objective function $F(X)$. The DPM (Back et al., 1991) deletes the infeasible solutions of the generated offspring after crossover and mutation operations, and reproduces the new offspring through crossover and mutation again, until all the generated offspring are feasible solutions. Michalewicz (1995) reported that the DPM can make search convergence in fewer generations, but with the price of spending longer time to produce feasible solutions. Thus, the real valued genetic algorithms (RGA) adopted in the research are classified into the RGA-SPM and the RGA-DPM with the algorithm flow charts shown in Fig. 6 and Fig. 7.

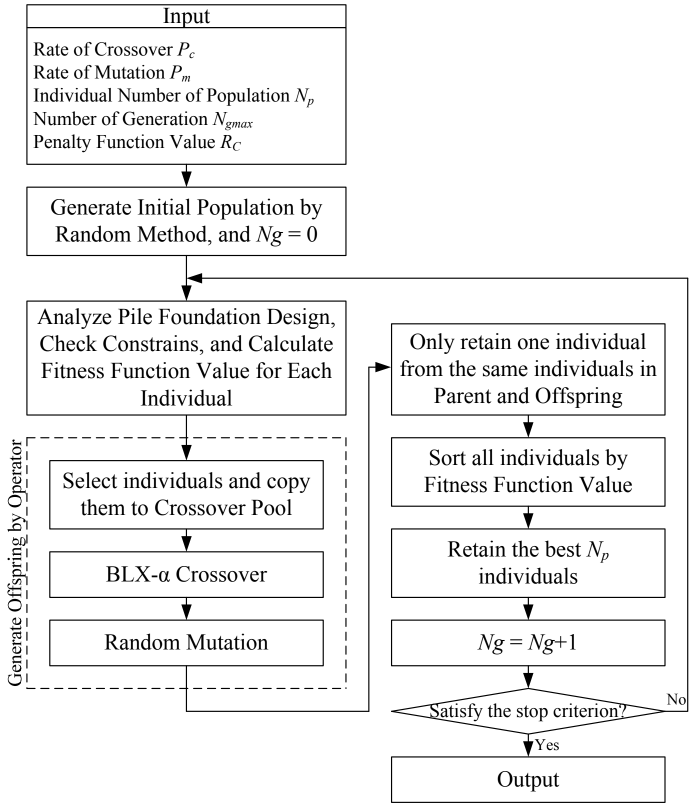

Fig.6 RGA-SPM flow chart

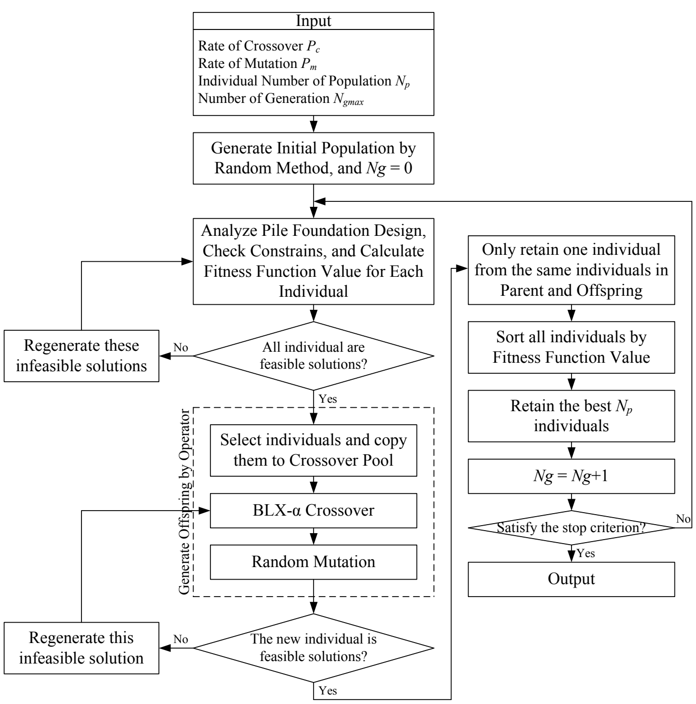

Fig. 7 RGA-DPM flow chart

B. Selection

The most used selection methods include roulette wheel selection, tournament selection, ranking selection and random selection methods. The former three methods use fitness values to choose more fit individuals into the crossover pool. Thus, the more fit the individuals, the more chance to be selected into the pool. This strategy is expected to achieve a more quick convergence since the better parents can produce the much better children, however, with the opportunity to reach premature convergence on poor solutions. For not missing the global optimal solution, this research uses the random selection method that is designed so that a small proportion of less fit solutions are selected. This helps keep the diversity of the population large, preventing premature convergence. The only parameter is the rate of crossover P_c. Each individual in the current population will have a random real number in [0, 1]. If the number is less than P_c, the associated individuals will enter into the crossover pool in sequence. The first individual enters in the pool will cross over with the one randomly chosen from the other individuals. The second individual will cross over in the same way until all the offspring are produced.

C. Crossover

The $BLX-\alpha'$ crossover method is used to perform crossover operation at the crossover point. The method extends the range of the i^{th} design variable $x_{P,i}$, the crossover point of two selected parent individuals, outward α' times of the original range, as shown in Fig. 8, thus redefine the range of the design variable of offspring individuals.

The α' parameter is set to be 0.25. If the i^{th} design variable $x_{P,i}$ is selected to be the crossover point, In Fig. 8, $P_{\max} = \max(x_{P1,i}, x_{P2,i})$, $P_{\min} = \min(x_{P1,i}, x_{P2,i})$ and $I = P_{\max} - P_{\min}$. Here defining the step of upper and lower neighboring points with $x_{P,i}$ is Δx_i. Using a random number r in $[0,1]$, the i^{th} design variables of the child individuals are

$$C_1 = P_{\max} - round\left(r\frac{(1+2\alpha)I}{\Delta x_i}\right)\Delta x_i \tag{14}$$

$$C_2 = P_{\min} + round\left(r\frac{(1+2\alpha)I}{\Delta x_i}\right)\Delta x_i \tag{15}$$

In which, C_1 and C_2 are the new values of the design variable x_i of the children individuals, respectively. The *round* is a round off mathematics operator.

For the other design variables, the simple crossover method is used. The formula for two child individuals can be written as

$$X_{C1} = (x_{P1,1}, x_{P1,2}, \cdots, C_1, x_{P2,i+1}, \cdots, x_{P2,n}) \tag{16}$$

$$X_{C2} = (x_{P2,1}, x_{P2,2}, \cdots, C_2, x_{P1,i+1}, \cdots, x_{P1,n}) \tag{17}$$

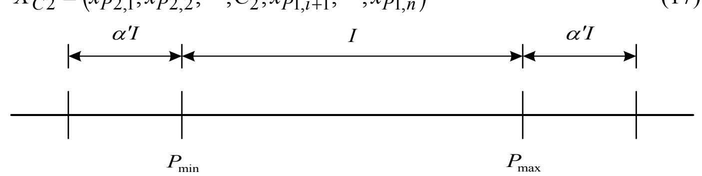

Fig. 8 BLX-α Crossover

D. Mutation

For simplicity and computational efficiency, the random mutation method is used to perform mutation operation. Here, the design variable selected to mutate is defined as x_M, $x_M \in [x_l, x_u]$ in which x_u, x_l are the upper and lower limits of x_M, the neighboring step is Δx_i. The mutated variable x_M^* can be generated by a random real number r in $[0,1]$ and written as below.

$$x_M^* = x_l + round\left(r \frac{x_u - x_l}{\Delta x_i} \right) \Delta x_i \quad (18)$$

In which, *round* is a round off mathematics operator.

Each design variable of the crossed over child individual has its own random number in $[0,1]$. If the number less than the rate of mutation P_M, the associated variable needs to mutate randomly. The only parameter is the rate of mutation P_M.

E. Choosing the Individuals of Child Population

After the selection, crossover and mutation, all the parent and child individuals are sorted in sequence based on their fitness values. The first N_P individuals which are more fit are chosen as new child population. To prevent premature, the same parent and child individuals are retained only one. This sorting is a kind of elitism strategy.

F. Stop Criteria

This generational process is repeated until some convergence criteria have been reached. Common criteria are:

(a) A solution is found that satisfies minimum criteria.
(b) Fixed number of generations N_{gmax} reached.
(c) Allocated budget (computation time/money) reached.
(d) The highest ranking solution's fitness has reached a plateau such that successive iterations no longer produce better results.

In the above criteria, (a) and (d) are difficult to judge and determine. The criterion (c) is actually similar to (b). Thus, this research uses a pre-set generation number N_{gmax} to terminate the computation based the results of parameter study.

4. Performance of ESM and RGA

In this section, six real design cases will be used to assess the performance of the ESM and RGA. Among them, cases I and II are cases of pile foundation designs for highway bridges. Cases III through VII are for Taiwan high speed railway (THSR) bridges. Tab. 2 shows the original design variables and the geological conditions of the seven cases. These data will used to compare the original design solution and the searched solutions of the ESM and RGA.

4.1 Performance of ESM

Tab. 3 shows the cost comparison of the original design solution and the ESM solution (regarded as the global optimal solution, GOS) for the seven cases. Obviously, the GOS gives a saving of about 17.6% to 57.6% as compared to the original design solution. However, from Tab. 3, the search time of the ESM is about 51,750 minutes (36 days). In general, the run time of the ESM can not be accepted in routine design practice.

Tab. 3 Comparison of the results searched by ESM and RGA for all cases

Case	Method	L_P	D	H_C	S_L	S_T	N_L	N_T	Cost (*NTD*)	Number of Analysis	Spending time (Min.)
Case I	Original	15	1.2	2.2	3.05	3.05	3	3	2,039,924	-	-
	ESM	26	1.1	1.8	4.10	3.60	2	2	1,391,274	10,221,120	51,100.0
Case II	Original	30	20	3.5	6.00	5.00	4	5	13,728,879	-	-
	ESM	15	1.5	3.80	4.30	3.80	4	6	6,681,762	11,943,936	59,720.0
Case III	Original	58	2.0	2.5	9.00	6.00	2	3	7,203,524	-	-
	ESM	25	1.8	2.8	4.50	4.60	4	2	4,541,702	10,349,856	51,750.0
Case IV	Original	58	2.0	2.5	9.00	6.00	2	3	7,203,524	-	-
	ESM	36	1.5	2.6	3.80	3.80	4	3	5,145,433	10,349,856	51,750.0
Case V	Original	52	2.0	2.5	8.00	8.00	2	2	4,614,478	-	-
	ESM	34	1.8	2.9	4.90	5.30	3	2	3,635,136	10,349,856	51,750.0
Case VI	Original	49	2.0	2.5	8.00	8.00	2	2	4,244,042	-	-
	ESM	25	2.0	2.9	5.00	5.00	3	2	3,497,323	10,349,856	51,750.0
Case VII	Original	50	2.0	2.5	8.00	8.00	2	2	4,296,826	-	-
	ESM	26	1.9	2.7	4.80	5.60	3	2	3,293,419	10,349,856	51,750.0

Tab. 2 Original Design Variables and Geological Descriptions for the studied cases

Case Name	Design Varibles							Geological Description	Has Bearing Layer? ($N \geq 50$)
	$L_P\,(m)$	$D\,(m)$	$H_C\,(m)$	$S_L\,(m)$	$S_T\,(m)$	N_L	N_T		
Case I	15	1.2	2.5	3.05	3.05	3	3	Interbedded layer (composed of the sandy layer and the clayey layer), and both are uniform distributions	Yes (16.2*m*)
Case II	30	2.0	3.5	6.00	5.00	4	5	Sandy layer. No clayey layer.	Yes (20.0*m*)
Case III	44	2.0	3.0	6.00	6.00	3	3	Interbedded layer (composed of the sandy layer and the clayey layer), and the majority is the sandy layer. Surface layer is a clayey layer. The maximum thickness of clayey layer is 6.8*m*.	No
Case IV	58	2.0	2.5	6.00	9.00	3	2	Interbedded layer (composed of the sandy layer and the clayey layer), and the majority is the sandy layer. The total thickness of three clayey layers is about 17.5*m*, and the maximum thickness of clayey layer is 12.2*m*.	Yes (46.0*m*)
Case V	52	2.0	2.5	8.00	8.00	2	2	Interbedded layer (composed of the sandy layer and the clayey layer), and the majority is the sandy layer. Surface layer is a clayey layer. The maximum thickness of clayey layer is 8.9*m*.	No
Case VI	49	2.0	2.5	8.00	8.00	2	2	Interbedded layer (composed of the sandy layer and the clayey layer), and the majority is the sandy layer. Surface layer is a clayey layer. The maximum thickness of clayey layer is 19.5*m*.	No
Case VII	50	2.0	2.5	8.00	8.00	2	2	Interbedded layer (composed of the sandy layer and the clayey layer), and the majority is the clayey layer. The total thickness of all sandy layers is about 12.2*m*, and the maximum thickness of sandy layer is 5.95*m*.	No

4.2 Performance of RGA

Since RGA is a random multiple-point search technique, the result of every RGA searching is different from the other. Thus, the RGA searching is executed one hundred times for each design case with some specific parameters, and the statistics of all the searched results are used to represent the performance of the RGA method. The performance of RGA will be influenced by the number of population N_P, the rate of crossover P_c, the rate of mutation P_m and the total number of generic generation $N_{g,\max}$. Therefore, at the first, Case I is used to conduct parameter study of the RGA algorithm. The test range of these four parameters is as below.

$$N_{g,\max} = 50,100,150,200$$

$$P_c = 0.7 - 0.9$$

$$P_m = 0.01 - 0.1$$

$$N_P = 5,10,20,30$$

Fig. 9 shows the decrease of the average cost of RGA solutions with the increase of generation number for different P_c and P_m with N_P=10. Based on the result, and considering both cost saving and run time, it is suggested that P_c=0.8, P_m=0.1, N_g=200. Using the above three parameters, the influence of N_P on the cost and the number of analysis is shown in Tab. 4. It shows that the larger the N_P, the lower the cost and the more the run time. Considering both cost saving and run time, N_P=20 is used in the following analysis.

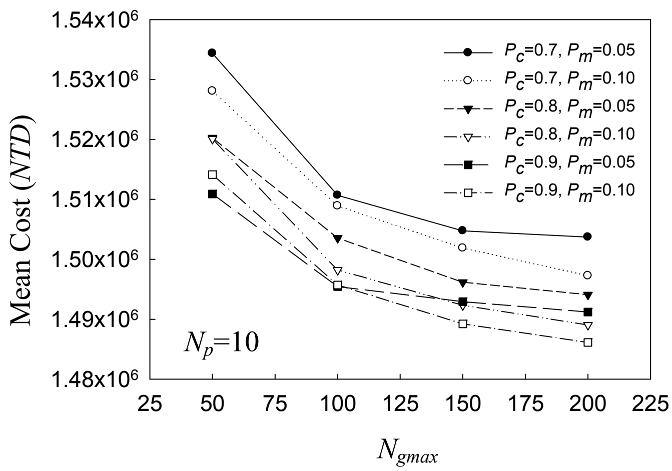

Fig. 9 Decrease of the average cost of RGA solutions with the increase of generation number for different P_c and P_m with N_P=10

Tab. 4 Comparison of RGA searching results for different N_P

N_p	Cost (*NTD*)				Pile Analysis Counts	The proportion of best solution (%)
	Mean	Best	Worest	STD		
5	1,528,102	1,427,735	1,689,314	62,931	1,600	0
10	1,489,032	1,391,274	1,644,183	56,069	3,204	5
20	1,459,222	1,391,274	1,584,811	35,692	6,400	9
30	1,450,872	1,391,274	1,571,590	34,848	9,597	15

After determination of the main optimization parameters of RGA, the design solutions of the seven cases searched by RGA-SPM and RGA-DPM are summarized in Tab. 5 and Tab. 6. From Tab. 5, the RGA-SPM algorithm gives a cost saving of about 15.63%-55.92%, in average 31.26%, as compared to the original design solutions. The average cost difference between the RGA-SPM solutions with global optimal solution (GOS) is only 2%. The search time of RGA-SPM is about 48 minutes which is far less than the time spent by ESM. In each case, the RGA-SPM always has opportunity to search out GOS. Especially in Case IV, each solution is GOS for the one hundred search executions.

Tab. 5 Performance of RGA-SPM

Case	Saving form Original Case (%)	The difference to best solution (%)	The proportion of best solution (%)	Pile Analysis Counts	Time Spent (Min.)
Case I	28.88	4.28	15	9,597	48
Case II	48.06	1.48	7	9,616	48
Case III	55.92	3.48	5	9,609	48
Case IV	28.57	0.00	100	9,630	48
Case V	19.71	1.92	35	9,596	48
Case VI	15.63	2.34	78	9,601	48
Case VII	22.08	1.66	12	9,602	48
Mean	31.26	1.99	36	9,607	48

Tab. 6 Performance of RGA-DPM

Case	Saving form Original Case (%)	The difference to best solution (%)	The proportion of best solution (%)	Pile Analysis Counts	Time Spent (Min.)
Case I	28.82	4.37	6	13,160	66
Case II	47.85	1.88	4	14,641	74
Case III	55.85	3.66	2	11,390	57
Case IV	28.57	0.00	100	8,852	44
Case V	20.04	1.50	46	13,630	68
Case VI	17.56	0.00	100	10,125	51
Case VII	22.01	1.76	8	13,021	65
Mean	31.53	1.62	38	12,117	61

From Tab. 6, The RGA-DPM algorithm gives a cost saving of about 17.56%-55.85%, in average 31.53%, as compared to the original design solutions. The average cost difference between the RGA-DPM solutions with global optimal solution (GOS) is only 1.62%. The search time of RGA-DPM is about 61 minutes which is 13 minutes longer than the time spent by RGA-SPM. In each case, the RGA-SPM always has opportunity to search out GOS. Especially in Case IV and VI, each solution is GOS for the one hundred search executions. The solution quality of RGA-DPM is a little better than that of RGA-SPM, but with the price of a little longer search time.

For the seven cases in the study, the one with the minimum cost among the one hundred solutions searched by RGA has to be GOS and the search time is only 48-61minutes, which is far less 51,750 minutes, the time spent by ESM. The above results demonstrate the performance of RGA method is very good in searching GOS of pile foundation design problem.

CONCLUSIONS

This paper presents a complete framework where pile foundation design can be optimized by RGA method for minimizing the total construction costs. The adopted design methodology is code-based so that it is quite acceptable in routine designs. The time of one hundred executions of RGA search is about one hour which is far faster than 36 days, the time spent by ESM to search global optimal solution. For the seven real case studies discussed in the paper, we demonstrate how reliably the RGA can find the global optimal solution during one hundred random search. The RGA solutions also save about 16% to

56% in total construction costs for the seven cases, as compared to the original design solutions. Thus, this methodology is shown to be a promising tool for solving optimization problems in the applicable geotechnical fields.

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